NEW CORRELATIONS FOR CONVECTIVE HEAT TRANSFER COEFFICIENT ON INDOOR FENESTRATION SURFACES—COMPILATION OF MORE RECENT WORK

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ABSTRACT

Recommendations for indoor fenestration surface natural convection heat transfer coefficients are given as a function of temperature difference, fenestration surface length, and inclination angle from horizontal. The angles of inclination cover both "winter" (surface heated from below) and "summer" (surface heated from above) conditions. The current indoor natural convection heat transfer correlations used in fenestration thermal calculation programs are based on outdated results, since they do not account for fenestration system length and are incomplete for inclination angles other than vertical (90 degrees) and horizontal (0 degrees and 180 degrees).

The proposed set of correlations is compiled from more recent results of several different experimental studies. The studies were carried out in the late 1960s and early 1970s by researchers in the United States and Japan. The subject of those studies was laminar and turbulent free convection over isothermal or constant heat flux surfaces. Different Prandtl number (Pr) fluids were used and heat transfer surfaces of different lengths were investigated for a range of inclination angles (Θ). Heat transfer correlations are presented in nondimensional form in terms of integrated Nusselt numbers (Nu) as a function of the Rayleigh number (Ra) and inclination angle (Θ). The transition between laminar and turbulent flow is given in terms of a critical Rayleigh number (Ra_c), which includes the dependence of Ra_c on Θ. The correlations from different studies agree quite well, despite differences in the experimental facilities, and represent improvements over results from the 1930s and 1940s that are still being used in building heat transfer calculations. The impact of the new set of correlations is studied for several typical fenestration system configurations and compared to heat transfer results using the old correlations.

The new set of correlations is proposed for inclusion in draft ASHRAE standard SPC142P, "Standard Method for Determining and Expressing Fenestration Heat Transfer." The proposed correlations could also be used to update the natural convection heat transfer correlations in Table 5, chapter 3, and the fenestration U-factor data in Table 5, chapter 27, of the ASHRAE Handbook—Fundamentals *(ASHRAE 1993).*

BACKGROUND

The current choice for the indoor surface heat transfer coefficient in North American fenestration system calculational procedures (ASHRAE 1993; EEL 1993; Finlayson et al. 1993; LBL 1993; UoW 1992) is based on the correlation for laminar natural convection on a vertical flat plate (Weise 1935; Saunders 1936), which is referenced in McAdams (1954):

$$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4};\ 10^4 < \mathrm{Ra}_L < 10^9. \tag{1}$$

A similar relationship is used in Table 5, chapter 3, in the *ASHRAE Handbook—Fundamentals* (ASHRAE 1993), with a slightly different constant (0.56) and upper limit (10^8), which appears to be a typographical error, since derived Equations 2a and 2b are identical to the ones in McAdams (1954). Equation 1 and the correlation for the turbulent regime are used for the derivation of "simplified equations for air at normal temperature" in McAdams (1954), which are

$$h = 1.42\left(\frac{\Delta T}{L}\right)^{1/4};\ 10^4 < \mathrm{Ra}_L < 10^9 \tag{2a}$$

and

$$h = 1.31\,(\Delta T)^{1/3};\ 10^9 < \mathrm{Ra}_L < 10^{12} \tag{2b}$$

where

ΔT = $|T_i - T_s|$, °C;
T_i = room air temperature, °C;
T_s = indoor fenestration surface temperature, °C;

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h = convective surface heat transfer coefficient, $W/m^2 \cdot K$; and
L = flat plate length, m.

It is interesting to determine the value of "normal temperature" used to derive Equations 2a and 2b. Surprisingly, the air film temperature is $T_f \approx 100°C$ ($\approx 210°F$), which is a much higher temperature than is typical for building indoor conditions (around 20°C [≈ 70°F]). In the outline for draft ASHRAE Standard 142 (ASHRAE 1990), heat transfer correlations for indoor natural convection heat transfer are given for vertical, horizontal, and 45-degree inclined indoor surfaces (see Equations 3a through 3e). These correlations were compiled from the methods used currently in one-dimensional fenestration heat transfer computer programs (LBL 1993; UoW 1992). Equation 3a was derived from Equation 2a, assuming the length of the surface to be $L \approx 0.41$ m (1.35 ft):

$$h = 1.77\Delta T^{1/4};\ \Theta = 90°\ (\Phi = 90°) \tag{3a}$$

$$h_b = 3.8353502C_{nor};\quad \Theta = 45°\ (\Phi = 45° \text{ and } \Phi = 135°) \tag{3b}$$

$$h_a = 2.2477496C_{nor};\quad \Theta = 135°\ (\Phi = 45° \text{ and } \Phi = 135°) \tag{3c}$$

$$h_b = 4.0054502C_{nor};\quad \Theta = 0°\ (\Phi = 0° \text{ and } \Phi = 180°) \text{ and} \tag{3d}$$

$$h_a = 0.9436496C_{nor};\quad \Theta = 180°\ (\Phi = 0° \text{ and } \Phi = 180°) \tag{3e}$$

where

Θ and Φ are defined in Figure 1,
a = heated from above,
b = heated from below,
C_{nor} = normalization coefficient,

$$C_{nor} = 0.582\Delta T^{1/4}.$$

It should be noted that the above heat transfer coefficients, even though developed from laminar heat transfer correlations, are independent of the fenestration system height (flat plate length), which is contrary to the heat transfer theory.

In building envelope heat transfer, two different design conditions are of importance: winter and summer. For each design condition, there are two different situations, (a) surfaces inclined above the vertical ($\Phi < 90°$) and (b) surfaces inclined below vertical ($\Phi > 90°$), giving four possible situations, as shown in Figure 1. Due to similarities in flow patterns, the four different situations can be reduced to two. The first is a surface heated from above ($\Theta > 90°$) and the second is a surface heated from below ($\Theta < 90°$). For the typical angles of inclination (i.e., $\Phi < 90°$), the two typical conditions are "summer" (Figure lc) and "winter" (Figure la).

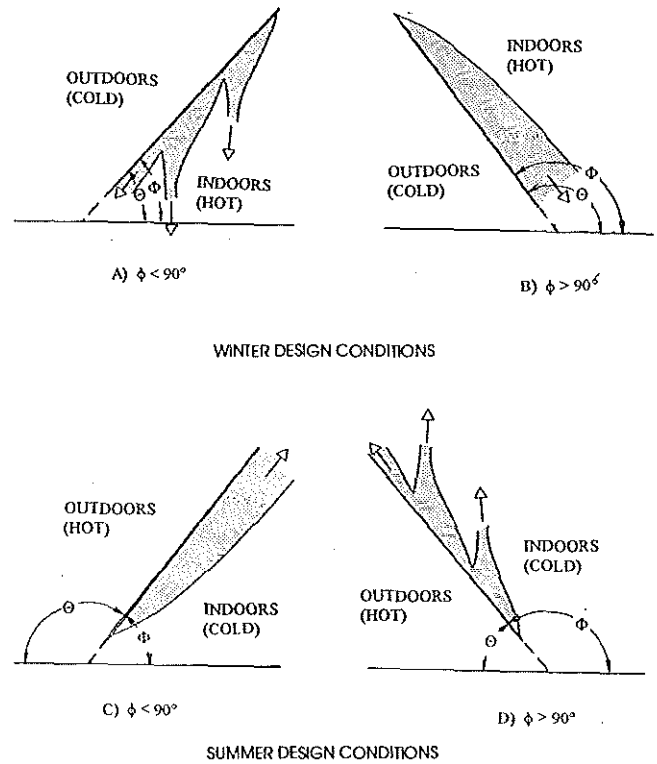

Figure 1 *Definition of inclination angles under winter and summer design conditions.*

Proposed Set of Correlations

The indoor glazing surface of a fenestration system, when subjected to realistic convective and radiative boundary conditions, behaves more like the flat plate under uniform heat flux (UHF) conditions than uniform temperature (UT) conditions. This was observed from the numerical heat transfer studies of fenestration systems by Curcija and Goss (1994), where the fenestration system was analyzed when subjected to variable convective surface heat transfer coefficients, while radiative heat transfer was modeled on the exterior surfaces and within the glazing cavity (see Figure 2). This set of boundary conditions simulated realistic heat transfer boundary conditions on both the outdoor (weather) and indoor (room) fenestration surfaces. From Figure 2 it can be observed that in the center-of-glass area the heat flux is fairly uniform, while the regions of the frame and edge-of-glass regions sharply depart from either UHF or UT conditions. In a study of the local convective heat transfer on indoor vertical fenestration surfaces, Curcija and Goss (1993) have shown that the value of the convective heat transfer coefficient in the regions of frame and edge of glass depends primarily on the geometry of the surface, due to the presence of frame steps.

In this paper, improved correlations based on a flat plate geometry are presented so the UHF boundary conditions for the center-of-glass region will be used in the selection of the proposed heat transfer correlations. As

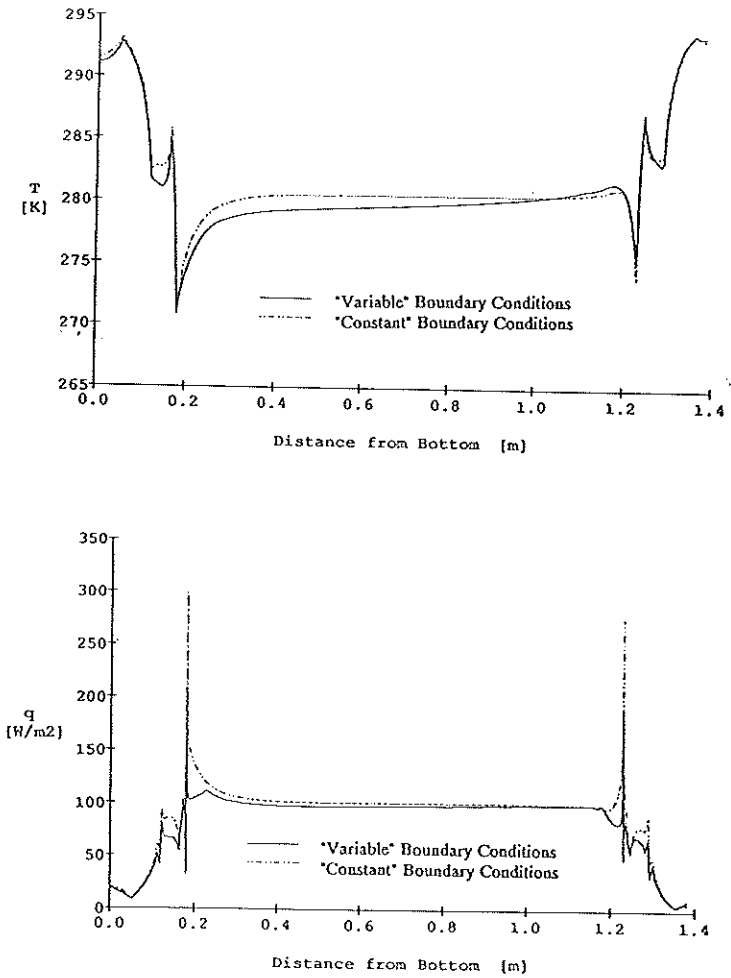

Figure 2 *Temperature and local heat flux distribution on indoor fenestration surfaces (taken from Curcija and Goss (1994)).*

previously indicated, UHF provides a more realistic condition for the indoor fenestration surface, so the results that are obtained for the UHF conditions will be used as a basis for the recommended correlations. The main difference between the results for the currently used UT conditions and the UHF conditions are in the transition region from laminar to turbulent flow, where the UT conditions give a critical Rayleigh number, Ra_c, which is approximately two orders of magnitude higher than for UHF conditions. Churchill and Chu (1975) found that the overall correlation for laminar and turbulent flow applied well for both boundary conditions, UHF and UT. The results for UHF conditions are usually reported in the literature in terms of Grashof number or Rayleigh number, which are based on heat flux rather than temperature difference (Gr^*, Ra^*). The conversion between the two is easily accomplished utilizing the Nusselt number, Nu:

$$Gr^* = \frac{g\beta q L^4}{v^2 k} = Gr \cdot Nu,$$

$$\text{or } Ra^* = \frac{g\beta q L^4}{v\alpha k} = Ra \cdot Nu$$

where * indicates uniform heat flux (UHF).

While the natural convection heat transfer correlation results for vertical and horizontal orientations of surfaces have been available for a long time (Weise 1935; Saunders 1936; Ostrach 1952; McAdams 1954), correlations for inclined surfaces were developed more recently. The first theoretical study was done by Rich (1953), who found that the correlations for vertical surfaces can be used for inclined surfaces if the gravitational term in Gr is altered to be the component parallel to an inclined surface. In later experimental studies by Vliet (1969) and Fujii and Imura (1972), this was confirmed for the laminar flow regime, and it was also concluded that in the turbulent flow regime the heat transfer coefficient is independent of the angle of inclination. Fujii and Imura (1972) covered a wide range of flat-plate tilt angles, including two horizontal orientations for both winter and summer conditions and for a vertical orientation of the plate. They used two electrically heated flat plates, 5 cm (2 in.) and 30 cm (1 ft) in length, immersed in water. They stated that their experimental conditions did not truly represent either UHF or UT conditions but said that their experimental "conditions are most practical." This observation basically agrees with the temperature and local heat flux distribution on the indoor side of a glazing surface (see Figure 2), which shows neither UHF nor UT conditions (although it is closer to UHF conditions). The results in the study of Fujii and Imura were reported for shorter and longer flat plates, and, in the case of $\Theta < 90°$, the results were different enough to warrant two different correlations (for different Ra, based on different lengths). In the set of correlations proposed here, the results for the longer plate were used, since 5 cm (2 in.) is too short for any realistic fenestration system.

Vliet (1969) found that the transition from the laminar to the turbulent flow regime for inclined surfaces exposed to "summer" conditions (with $\Theta < 90°$) depends on the angle of inclination, and from his results a relation was developed that correlates Ra_c and Θ (Equation 4b-3).

The recommended set of correlations for all angles of inclination, as defined in Figure 1, is derived from the work of Fujii and Imura (1972), except for Equation 4b-3, which is derived from the work of Vliet (1972); all are given here. For clarity, only the Rayleigh number based on temperature difference (Ra) was used.

A. Surfaces inclined from 0° to 15° ($0° \le \Theta \le 15°$)

$$Nu = 0.13 Ra_L^{1/3}; \text{ for all } Ra_L. \tag{4a}$$

B. Surfaces Inclined from 15° to 90° ($15° \le \Theta \le 90°$)

$$Nu = 0.56\,(Ra_L \sin\Theta)^{1/4}; \; Ra_L \le Ra_c \tag{4b-1}$$

$$\mathrm{Nu} = 0.13\,(\mathrm{Ra}_L^{1/3} - \mathrm{Ra}_c^{1/3}) + 0.56\,(\mathrm{Ra}_c \sin\Theta)\,;\ \mathrm{Ra}_L \geq \mathrm{Ra}_c \tag{4b-2}$$

where

$$\mathrm{Ra}_c = 2.5 \times 10^5 \left(\frac{e^{0.72\Theta}}{\sin\Theta}\right)^{1/5}\,; \tag{4b-3}$$

Θ is in degrees.

C. Surfaces Inclined from 90° to 179° ($90° < \Theta \leq 179°$)

$$\mathrm{Nu} = 0.56\,(\mathrm{Ra}_L \sin\Theta)^{1/4};\ 10^5 \leq \mathrm{Ra}_L \leq 10^{11}. \tag{4c}$$

D. Surfaces at $\Theta = 180°$

$$\mathrm{Nu} = 0.58\mathrm{Ra}_L^{1/5};\ 10^6 \leq \mathrm{Ra}_L \leq 10^{11}. \tag{4d}$$

Note: The physical properties for all of these correlations are evaluated at an equivalent film temperature, defined as

$$T_f = T_i - 0.25\,(T_i - T_s).$$

Using the physical property data at an indoor film temperature of T_f = 15°C (≈ 60°F) and also recognizing that $h = (k/L)\mathrm{Nu}$, Equations 4a through 4d become, for the heat transfer coefficient, h (using the SI system of units):

A. Surfaces Inclined from 0° to 15° ($0° < \Theta < 15°$)

$$h = 1.59\Delta T^{1/3};\ \text{for all } \mathrm{Ra}_L. \tag{5a}$$

B. Surfaces Inclined from 15° to 90° ($15° \leq \Theta \leq 90°$)

$$h = 1.46\left(\frac{\Delta T}{L}\sin\Theta\right)^{1/4};\ \mathrm{Ra}_L < \mathrm{Ra}_c \tag{5b-1}$$

$$h = 1.59\Delta T^{1/3} - 0.13\frac{k}{L}\mathrm{Ra}_c^{1/3} + 0.56\frac{k}{L}(\mathrm{Ra}_c \sin\Theta)^{1/4};\ \mathrm{Ra}_L \geq \mathrm{Ra}_c \tag{5b-2}$$

where

$$\mathrm{Ra}_c = 2.5 \times 10^5 \left(\frac{e^{0.72\Theta}}{\sin\Theta}\right)^{1/5}. \tag{5b-3}$$

Θ is in degrees.

C. Surfaces Inclined from 90° to 179° ($90° < \Theta < 179°$)

$$h = 1.46\left(\frac{\Delta T}{L}\sin\Theta\right)^{1/4};\ 10^5 \leq \mathrm{Ra}_L \leq 10^{11}. \tag{5c}$$

D. Surfaces at $\Theta = 180°$

$$h = 0.6\left(\frac{\Delta T}{L}\right)^{1/5};\ 10^6 \leq \mathrm{Ra}_L \leq 10^{11}. \tag{5d}$$

RESULTS OF COMPARISONS

Tables 1, 2, and 3 present, for a wood casement framed window of different heights and insulated glazing units (IGU), the indoor natural convection heat transfer coefficient, h_{ic}, and the overall U-factor, U. The values were calculated using one of the programs and the new set of recommended correlations given in Equations 4a through 4d ("recomm"). U-factors for the "recomm" column were calculated using the program's algorithm but with the current h_{ic} values from the "W4.1" column stripped off and replaced with the recommended h_{ic} values from the "recomm" column. Equations 6a through 6c illustrate this procedure.

Table 1 **Comparison of Indoor Natural Convection Heat Transfer Coefficients and Overall U-Factors for Wood Casement Windows with ASHRAE Winter Conditions and Vertical Orientation (Θ = 90°) – SI Units (W/m²·K)**

Fen. Sys. Height (in.)	IGU Configuration															
	Single				Double				Double, Low-e				Triple, Low-e			
	W4.1		recomm		W4.1		recomm		W4.1		recomm		W4.1		recomm	
	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U
24	4.13	5.04	3.90	4.95	3.41	2.80	3.19	2.75	3.08	2.19	2.87	2.17	2.83	2.06	2.65	2.05
48	4.13	5.04	3.28	4.71	3.41	2.80	2.68	2.68	3.08	2.19	2.42	2.13	2.83	2.06	2.23	2.01
60	4.13	5.04	3.10	4.64	3.41	2.80	2.54	2.66	3.08	2.19	2.29	2.12	2.83	2.06	2.11	2.00
72	4.13	5.04	2.96	4.58	3.41	2.80	2.42	2.64	3.08	2.19	2.18	2.11	2.83	2.06	2.01	2.00
82	4.13	5.04	2.87	4.55	3.41	2.80	2.35	2.63	3.08	2.19	2.11	2.10	2.83	2.06	1.95	1.99

Table 2 **Comparison of Indoor Natural Convection Heat Transfer Coefficients and Overall U-Factors for Wood Casement Windows with ASHRAE Winter Conditions and Sloped Orientation (Θ = 45°) – SI Units (W/m²·K)**

Fen. Sys. Height (in.)	IGU Configuration															
	Single				Double				Double, Low-e				Triple, Low-e			
	W4.1		recomm		W4.1		recomm		W4.1		recomm		W4.1		recomm	
	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U
24	5.17	5.45	4.43	5.19	4.31	3.00	3.28	2.87	4.00	2.46	2.89	2.36	3.68	2.07	2.52	1.99
48	5.17	5.45	4.71	5.29	4.31	3.00	3.56	2.91	4.00	2.46	3.18	2.39	3.68	2.07	2.81	2.01
60	5.17	5.45	4.76	5.31	4.31	3.00	3.62	2.92	4.00	2.46	3.24	2.39	3.68	2.07	2.86	2.02
72	5.17	5.45	4.80	5.32	4.31	3.00	3.65	2.92	4.00	2.46	3.27	2.40	3.68	2.07	2.90	2.02
82	5.17	5.45	4.82	5.33	4.31	3.00	3.67	2.92	4.00	2.46	3.30	2.40	3.68	2.07	2.92	2.02

Table 3 **Comparison of Indoor Natural Convection Heat Transfer Coefficients and Overall U-Factors for Wood Casement Windows with ASHRAE Winter Conditions and Horizontal Orientation (Θ = 0°) – SI Units (W/m²·K)**

Fen. Sys. Height (in.)	IGU Configuration															
	Single				Double				Double, Low-e				Triple, Low-e			
	W4.1		recomm		W4.1		recomm		W4.1		recomm		W4.1		recomm	
	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U	h_{ic}	U
24	5.39	5.53	5.03	5.41	4.56	3.17	3.91	3.09	4.30	2.68	3.61	2.61	3.83	2.08	3.08	2.03
48	5.39	5.53	5.03	5.41	4.56	3.17	3.91	3.09	4.30	2.68	3.61	2.61	3.83	2.08	3.08	2.03
60	5.39	5.53	5.03	5.41	4.56	3.17	3.91	3.09	4.30	2.68	3.61	2.61	3.83	2.08	3.08	2.03
72	5.39	5.53	5.03	5.41	4.56	3.17	3.91	3.09	4.30	2.68	3.61	2.61	3.83	2.08	3.08	2.03
82	5.39	5.53	5.03	5.41	4.56	3.17	3.91	3.09	4.30	2.68	3.61	2.61	3.83	2.08	3.08	2.03

$$R_{w4.1} = \frac{1}{U_{w4.1}}; \tag{6a}$$

$$R_{mod} = R_{w4.1} - \left(\frac{1}{h_{ic}}\right)_{w4.1} + \left(\frac{1}{h_{ic}}\right)_{recomm}; \tag{6b}$$

$$U_{recomm} = \frac{1}{R_{mod}}. \tag{6c}$$

CONCLUSIONS AND RECOMMENDATIONS

The new recommended set of correlations ("recomm") gives somewhat lower indoor natural convection heat transfer coefficients (h_{ic}) and correspondingly lower U-factors than values calculated using the current procedure in ASHRAE and the program's model ("W4.1"). This is primarily due to the current use of heat transfer correlations that are based on small flat plate results that are then used in calculating heat transfer in fenestration systems that are several times larger. The currently used set of correlations therefore provides higher convective heat transfer coefficients and artificially increases the predicted overall heat transfer in fenestration systems.

The effects of the differences between the "W4.1" and "recomm" h_{ic} values are more pronounced for single-glazed fenestration systems since h_{ic} produces the primary resistance to the overall heat transfer rate. Also, the difference between the "W4.1" and "recomm" values increases for increasing fenestration system height for all investigated sizes at vertical orientation, while it goes in the opposite direction for sloped surfaces (e.g., Θ = 45°). This is primarily due to the presence of a mixed (part laminar and part turbulent) flow regime at sloped angles when Θ < 90° and Φ < 90° ("winter" conditions). At the horizontal orientation (e.g., Θ = 0°) for "winter" conditions, the heat transfer coefficient is constant with respect to fenestration system height due to the canceling effects of the 1/3 power characteristic for turbulent flows and L^3 in the Gr and Ra. Differences between the correlations are the least pronounced for the horizontal orientation due to fully turbulent flow over the complete fenestration surface. The differences between the U-factors listed in the "W4.1" and "recomm" columns range as follows: in Table 1, for vertical orientation, from 9.7% (single; 82 in.) to 0.5% (triple, low-e; 24 in.); in Table 2, for 45° sloped orientation, from 4.8% (single; 82 in.) to 2.4% (triple, low-e; 82 in.); and in Table 3, for horizontal orientation, are fairly uniform at about 2%.

The new proposed set of correlations for indoor natural convection heat transfer are based on more recent work, and they correctly incorporate fenestration system height as an additional variable. It is therefore recommended that they be used in the next generation of fenestration system heat transfer computer programs as well as in the proposed ASHRAE standard calculational method. They are also recommended for use in the new edition of the *ASHRAE Handbook—Fundamentals*, to be published in 1997.

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